Alphamagic Squares

Adventures with turtle shell and yew between the mountains of mathematics and the lowlands of logology.

by Lee C. F. Sallows

"Eleven + two = twelve + one" —Martin Gardner

The following article made its debut as a talk given this summer at the unique Eugene Strens Memorial Conference on Intuitive and Recreational Mathematics and Its History, *held at the University of Calgary, Alberta. The conference was designed to mark the University library's acquisition of the Strens Collection.*

Strens, whose home was at Breda in the south of Holland, devoted most of his life to collecting. On his death he left behind him what is probably the world's most remarkable assemblage of books on recreational math topics.

The author, who lives in the Netherlands and "stumbled across this Aladdin's Cave while exploring the Dutch mountains," contacted Martin Gardner about it. Together, they were instrumental in putting the Strens family in contact with Richard Guy of the Department of Mathematics at the University of Calgary, which eventually led to the collection being moved to its new home.

The history of *magic squares* is a venerable one, reaching back into the legendary past of ancient China. So it is that the simplest, oldest, and most famous square of all, the so-called *Lo shu (shu* meaning *writing, document),* is said to have first been revealed on the shell of a sacred turtle which appeared to the mythical Emperor Yü from the waters of the Lo river in the 23rd century B.C. (This is discussed in Camman's

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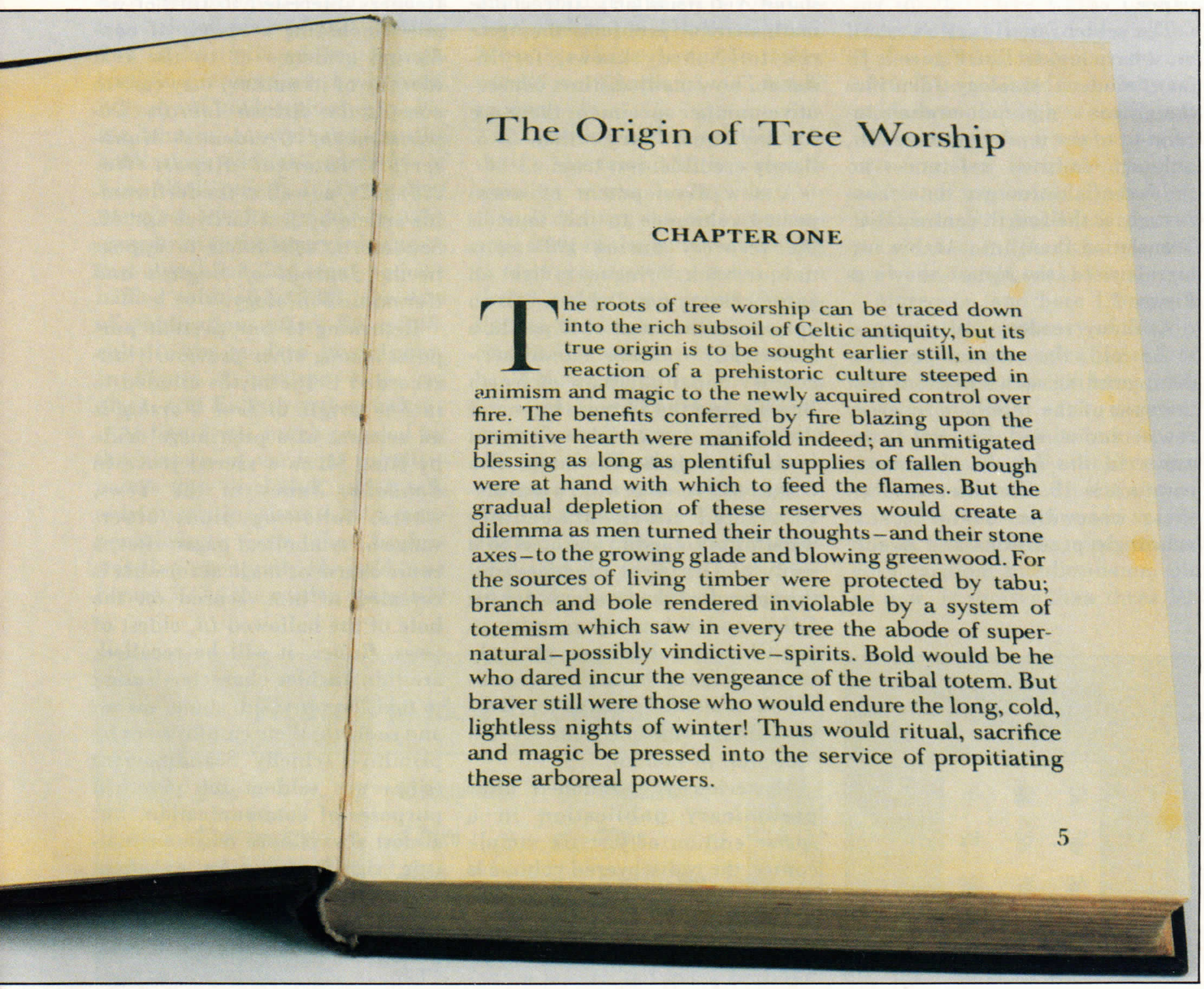
The opening page from *The Origin of Tree Worship*, a work shrouded in mystery. Privately published in England, 1887, a copy was placed in the British Museum but disappeared soon after. Now rediscovered after 98 years, its pages reveal—among other riddles—the unprecedented *Li shu*, a mathematico-linguistic formula of demonstrably magical power. [Courtesy of the British Library, London.]



The Origin of Tree Worship

CHAPTER ONE

The roots of tree worship can be traced down into the rich subsoil of Celtic antiquity, but its true origin is to be sought earlier still, in the reaction of a prehistoric culture steeped in animism and magic to the newly acquired control over fire. The benefits conferred by fire blazing upon the primitive hearth were manifold indeed; an unmitigated blessing as long as plentiful supplies of fallen bough were at hand with which to feed the flames. But the gradual depletion of these reserves would create a dilemma as men turned their thoughts–and their stone axes–to the growing glade and blowing greenwood. For the sources of living timber were protected by tabu; branch and bole rendered inviolable by a system of totemism which saw in every tree the abode of supernatural–possibly vindictive–spirits. Bold would be he who dared incur the vengeance of the tribal totem. But braver still were those who would endure the long, cold, lightless nights of winter! Thus would ritual, sacrifice and magic be pressed into the service of propitiating these arboreal powers.

5

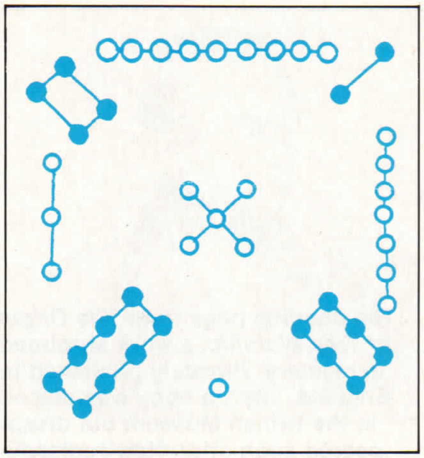


Figure 1. **The *Lo shu*, revealed on the shell of a sacred turtle, according to Chinese legend.**

article cited at the end of this paper.)

The celebrated turtle's shell must have looked like Figure 1. In fact, modern sinology identifies these signs as a pseudo-archaic invention of the tenth century A.D., although indirect references to the essential structure date from as early as the fourth century B.C. Translating them into Arabic numerals yields the square shown as Figure 2.

As many readers will not need to be told, the magical property distinguishing such squares is that the sum of the numbers in every row, column, and diagonal is the same; in the *Lo shu* this magic constant is 15. The literature on these recondite curiosities is amazingly prodigious and deeply ramified, not to say widely and haphazardly dispersed. It is clear that the spell cast by the elegant symmetries reflected in these interlocking number patterns has held countless devotees in thrall, eminent mathematician and lowliest layman alike. Hardly a turtle shell has been left unturned in exploring variations on the central theme, so that articles and even books abound devoted to special categories of squares, as well as magic triangles, rectangles, circles, stars, antimagic squares, prime-number squares, multiplicitively magic squares, magic cubes, N-dimensional arrays, and so on. Not least in adding spice to the subject is the variety of simply stated yet peculiarly intractable mathematical problems they give rise to. Nobody knows, for instance, how many distinct consecutive-number specimens there are for any square larger than 5×5. Barely credible, but true!

Translation of Figure 1

4	9	2
3	5	7
8	1	6

Figure 2

A new development of unexpected relevance to this topic is the recovery during 1985 of a unique book, bringing to light an extraordinary parallel between an episode in the reign of King Mī, a historically dubious late-fifth(?)-century tribal chieftain of North Britain, and the Chinese legend of the *Lo shu*. Apparently misplaced in or about 1888, this book, *The Origin of Tree Worship*, a privately printed nineteenth-century work of scholarship devoted to a study of Druidical practices and the spread of the yew cult among Celtic and Germanic peoples in pre-Christian Europe, recently surfaced again during a reorganization of bookshelves at the British Library (formerly the British Museum) in London.

Mysteriously abandoned after preliminary publication in a sparse edition of just six sample copies, the rediscovered volume is in all probability the only surviving exemplar (see photo), and its reappearance after nearly one hundred years has caused a considerable ripple in philological circles. The reason for this lies in a wealth of unmistakable internal evidence showing that the author must have been borrowing from medieval manuscript material previously believed lost in the fire that destroyed so much of the famous Cottonian collection of priceless early English documents, while it was housed at Little Deans Yard, Westminster, in 1731. As such, *The Origin of Tree Worship* is presently the subject of minute scrutiny by experts and, quite apart from the urgent questions thrown up by the provenance of its cited material, is already shedding light in several areas of paleographical research. Readers interested in further details (including a review of conflicting evidence as to the real identity of its author) may care to consult the *British Library Department of Occidental Manuscripts Internal Report No. 2704/1729*, as well as the forthcoming article by J. Allardyce and M. Sandeford, scheduled to appear in the *Journal of English and Germanic Philology*.

Returning to our present purpose: among other previously unrecorded Celtic myths alluded to in *The Origin of Tree Worship* is an account of a pilgrimage made by King Mī to a sacred grove in *Ēohdalir*, Valley of the Yews, where, following pious observance of symbolical pagan rites, a runic charm or magical formula is revealed to him, scored on the bole of the hallowed *Li*, eldest of yews. Runes, it will be recalled, are thin angular characters suited to incision on wood, stone, metal, and so forth; their employment by primitive (chiefly Scandinavian) tribes was seldom for practical purposes of communication, but almost always bore magico-ritualistic significance. An excellent survey of the subject is *Runes: An Introduction* by R. W. V. Elliott (Manchester University Press, 1980).

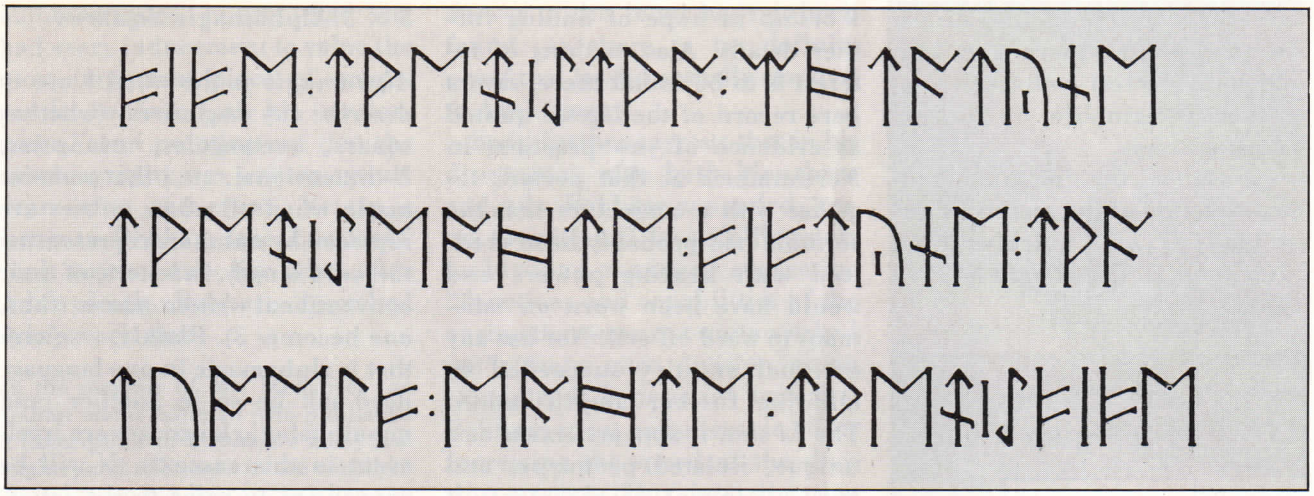


Figure 3. **The runic charm revealed to King Mī, scored on the bole of *Li*, eldest of the sacred yew trees.**

As an amateur runologist fortunate enough to have been granted a privileged view of this exciting find (a facsimile edition is presently in preparation), I was naturally drawn to deciphering the runic charm reproduced in the book along with the narrative of King Mī (Figure 3). It is a mark of the great advances made in paleography over the intervening years that a problem that seems to have baffled solution in 1887 (the date of publication) offers little difficulty to the modern investigator. In Figure 4, modern usage replaces Old English orthography.

At first I was much puzzled by the pattern of cardinal number-names thus disclosed, and it was only on writing them out in more perspicuous form that understanding eventually dawned (see Figure 5). As the reader can easily verify, the sum of the three elements occurring in every row, column, and diagonal is the same: 45. What we have here, in other words, is a familiar 3 × 3 *magic square*.

Fascinating as this parallel with the *Lo shu* legend is, however, it remains worth noting that although distinct, the nine numbers appearing in the runic square fail to form a consecutive series, as in their Chinese counterpart. Nevertheless, the *Li shu* (as I suppose it can hardly otherwise be called) bears closer examination. Seeking for something to warrant a supernatural manifestation on a sacred yew tree, and having already been prompted through registering a small coincidence while transliterating the runes, I soon discovered that the *number of runes*—and, thus, by chance, the number of *modern English letters*—making up the three words used in every row, column, and diagonal is also identical: there are twenty-one! (The coincidence between modern and archaic word lengths will seem of less moment to readers familiar with the normal course of etymological development from Old English forms: two = *twā*, five = *fifé*, eight = *eahte*, twelve = *tuoelf*, and so on).

Moreover (and here I began to appreciate the potency of this singular thaumaturgical device), writing out the rune or letter totals associated with each number-name not only results in a second magic square, the numbers now emerging do indeed comprise an unbroken consecutive series (Figure 6). Furthermore, since no English cardinal number-name, old or new, is shorter than three let-

Figure 3 in Modern English

Five	:	Twenty-two	:	Eighteen
Twenty-eight	:	Fifteen	:	Two
Twelve	:	Eight	:	Twenty-five

Figure 4

. . . and in Numerals

5	22	18
28	15	2
12	8	25

Figure 5

ters—the smallest number occurring here—this square even embodies the lowest consecutive sequence imaginable on purely lexical grounds.

Astonished by this unlooked-for revelation of the secrets inherent in the *Li shu*, I quickly turned back again to *The Origin of Tree Worship* in hope of finding further details. Alas, nothing of interest is to be found there, save a bare record of the legend quoted as evidence of yew practices in Northumbria at that period, together with a conjecture that the formula had probably been credited with healing powers and would have been worn on talismans to ward off evil. Nor has any external enquiry succeeded in eliciting further amplification. The *Li shu*, it appears, exists as a unique, isolated prototype, and any subsequent developments it may have given rise to have long since been lost to us, buried in the dust of history.

Letter Counts from Figure 4

4	9	8
11	7	3
6	5	10

Figure 6

General Formula

$a+b$	$a-b-c$	$a+c$
$a-b+c$	a	$a+b-c$
$a-c$	$a+b+c$	$a-b$

Figure 7

Obscure as its origins remain, clearly the rediscovery of this fantastic formula immediately provokes a host of tantalizing questions and contingencies quite independent of the historical, mythological, philological, and, indeed, criminological issues raised in connection with *The Origin of Tree Worship* itself. In fact, as the following will show, the *Li shu* furnishes a point of departure into an exciting new genre, a hitherto undreamed-of field, perhaps best described as a kind of recreational department of Computational Linguistics. I refer to the exploration of *alphamagic squares.*

3 × 3 Alphamagic Squares

Alphamagic is the word I use to describe any magic array (whether square, rectangular, triangular, *N*-dimensional, etc.) that remains magic when all of its entries are replaced by numbers representing the word length, in letters, of their conventional written names (thus, *one* becomes 3). Plainly, a square that is alphamagic in one language need not be so in another (and nonalphabetic languages are irrelevant in this context). It will be convenient to refer to the letter-count of a number-word as the *logorithm*—or *log*, for short—of the original number (*logos* = word, *arithmos* = number). Logorithm should not be confused with a similar word coined by a Scotchman called Napier in 1614. Where unstipulated, "natural" logs or $\log_{e(nglish)}$ will be assumed; hence *log* 15 + *log* 3 = log_{french} 69 since 7 + 5 = 12, the number of letters in *soixante-neuf.*

By *magic* I shall mean any arrangement producing a constant sum along its various orthogonals and diagonals, regardless of whether the elements involved are distinct or not. Naturally, a square showing repeated entries is less interesting than one in which all are different. The *order* of a square refers to its size—the num-

A Table of Natural Logs

numbers	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
logorithms	*4*	*3*	*3*	*5*	*4*	*4*	*3*	*5*	*5*	*4*	*3*	*6*	*6*	*8*	*8*	*7*	*7*	*9*

numbers	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	. . .
logorithms	*8*	*8*	*6*	*9*	*9*	*11*	*10*	*10*	*9*	*11*	*11*	*10*	*6*	*9*	*9*	*11*	*10*	*10*	*. . .*

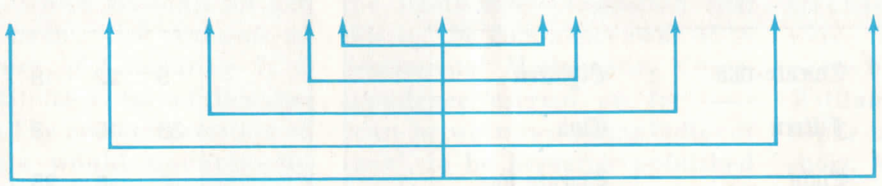


Figure 8. **The natural numbers 0 through 35 together with their logorithms. Connecting lines indicate triples in which both numbers and logorithms form regular arithmetic series.**

ber of cells on a side. The *Li shu* is thus an English alphamagic square of order 3, having as additional properties that its logorithms are distinct, consecutive, and *minimal* (that is, they comprise a set of the smallest possible non-alike logorithms existent in English). Order-1 and order-2 squares need not detain us, as a moment's consideration will show. With these few conventions established, we are ready to pursue the main theme.

As a tentative entry into the unfamiliar terrain, it is natural to wonder if there are any 3×3 alphamagic squares other than the one produced above. Useful in this connection is the general formula for order 3 shown in Figure 7 (and due to Édouard Lucas), since both numbers and logorithms in an alphamagic square must satisfy the relations it exemplifies.

Note that the three elements on each straight-line bisector through the center form a set of four 3-term arithmetic series (that is, they show a constant difference between adjacent terms: $[a-b+c]-a = a-[a+b-c]$, for instance). Then one obvious initial step is to search for arithmetic triples whose logorithms share the same property.

Figure 8 lists the cardinal numbers from 0 to 35 together with their English logorithms. Taking for illustration a center number C of 15, consider in turn arithmetic triples formed by C and its equidistant neighbors $C-1$ and $C+1$, $C-2$ and $C+2$, and so on. Note down those cases in which $log(C-N)$, $log\ C$ $(=7)$, and $log(C+N)$ also form arithmetic triples. When $N = C$ we can go no further, since $C - N = 0$. By now we shall have a list of pairs of associated arithmetic triples (Figure 9).

If there are any 3×3 alphamagic squares with a center number of 15 (and we already know there is one), at least *four* of these five cardinal-number triples must appear in it: one along each straight-line bisector, including diagonal bisectors.

Selecting now the first two triple-pairs on the list for closer scrutiny, write them into the diagonals of corresponding matrices (Figure 10). The choice of *diagonals* here is not critical; alternative linear cell-groups might be used. We argue that since the latter will have to be occupied by two of the listed cardinal-number triples, testing each pair in turn in these positions will comprise an exhaustive check of all possibilities. Note that changing the order in which a given pair is written into the diagonals merely creates rotations or reflections of the same configuration.

Referring back to the general formula, we find that the magic constant of any square is always 3

Cardinal Numbers / **English Logorithms**

12	15	18	:	6	7	8
11	15	19	:	6	7	8
8	15	22	:	5	7	9
5	15	25	:	4	7	10
2	15	28	:	3	7	11

Figure 9

Cardinal Numbers

12		11
	15	
19		18

English Logorithms

6		6
	7	
8		8

Figure 10

Li Shu

fundamental

5	22	18
28	15	2
12	8	25

second harmonic

105	122	118
128	115	102
112	108	125

Figure 11

ALPHA.BAS

```
10 ' ***********************************************************************
20 ' *    Program:  ALPHA.BAS       (GWBASIC)                              *
30 ' *    Purpose:  To generate and print out all 3 × 3 alphamagic         *
40 ' *              squares formable using 9 distinct cardinals in         *
50 ' *              the range 0 - NMAX. Index numbers and logorithm        *
60 ' *              squares are printed alongside.                         *
70 ' *    Author:   Lee C.F. Sallows                                       *
80 ' *    Date:     Guy Fawkes Day (November 5th), 1985                    *
90 ' ***********************************************************************
100 ' _____________________________________________________________________
110 '      Array definitions:
120 '      A        Logorithms of 0 - NMAX (loaded as data)
130 '      B        Arithmetic triples showing arithmetic logorithms
140 '      C        Logorithm-triple counterparts to numbers in B
150 ' _____________________________________________________________________
160    NMAX=109          'In this example
170    DIM A(NMAX):   DIM B(100,3):   DIM C(100,3)
180 ' _____________________________________________________________________
190 '  Data is loaded into A; A(n) thus corresponding to log n.
200 '  Contingent triple CENtre numbers are considered in turn.
210 ' _____________________________________________________________________
220    FOR I=0 TO NMAX:   READ A(I):   NEXT I
230    FOR CEN = 4 TO NMAX-4
240 ' _____________________________________________________________________
250 '  Starting with the two highest-lowest values possible (BOUND)
260 '  and working inwards, A-elements equidistant about A(CEN) are
270 '  checked with A(CEN) to see if they form arithmetic triples. If so,
280 '  COUNT is incremented, the number-triple is stored in B and its
290 '  associated log-triple stored in C.
300 '  Provided at least 4 triple-pairs are found we proceed to the next
310 '  stage; otherwise reset COUNT and take the next CENtre number.
320 ' _____________________________________________________________________
330    IF CEN(NMAX-CEN) THEN BOUND=CEN ELSE BOUND=(NMAX-CEN)
340    IF A (CEN)-A(CEN-BOUND) <>A(CEN+BOUND)-A(CEN) THEN GOTO 380
350    COUNT=COUNT+1
360    B(COUNT,1)=(CEN-BOUND): B(COUNT,2)=CEN: B(COUNT,3)=(CEN+BOUND)
370    C(COUNT,1)=A(CEN-BOUND): C(COUNT,2)=A(CEN): C(COUNT,3)=A(CEN+BOUND)
380    BOUND=BOUND-1: IF BOUND<>1 THEN GOTO 340
390    IF COUNT<4 THEN GOTO 960
400 ' _____________________________________________________________________
410 '  B now contains 4 or more arithmetic triples, C their associated
420 '  logorithms. Using I and J to address every possible pair of
430 '  B-triples in turn, we deal with them as though written into the
440 '  diagonals of a 3 × 3 test matrix, thus:
450 '                  ______________________
460 '                   B(I,1)   .   B(J,3)
470 '                  ______________________
480 '                     .    CEN     .
490 '                  ______________________
500 '                   B(J,1)   .   B(I,3)
510 '                  ______________________
520 '  Magic-fulfilling values are now calculated for the remaining empty
530 '  cells, checking one at a time to see if their logorithms also
540 '  satisfy magic conditions in the associated log-matrix.
550 ' _____________________________________________________________________
560    FOR I=1 TO COUNT-1
570    FOR J=I+1 TO COUNT
580    CL=3*CEN-B(I,1)-B(J,1)          'CL = centre left column number
590    IF CL>NMAX OR CL <0 THEN GOTO 950 'entries must be within limits
600    IF A(CL) <>3*A(CEN)-C(I,1)-C(J,1) THEN GOTO 950 'check left column log.
```

```
610      CT=3*CEN-B(I,1)-B(J,3)    'CT = centre top row number
620      IF CT>NMAX OR CT<0 THEN GOTO 950   'within limits?
630      IF A(CT) <>3*A(CEN)-C(I,1)-C(J,3) THEN GOTO 950 'check top row log.
640      CR=3*CEN-B(I,3)-B(J,3)    'CR = centre right column number
650      IF CR>NMAX OR CR<0 THEN GOTO 950   'within limits?
660      IF A(CR) <>3*A(CEN)-C(I,3)-C(J,3) THEN GOTO 950 'check right column log.
670      CB=3*CEN-B(I,3)-B(J,1)    'CB = centre bottom row number
680      IF CB>NMAX OR CB<0 THEN GOTO 950   'within limits?
690      IF A(CB) <>3*A(CEN)-C(I,3)-C(J,1) THEN GOTO 950 'check bottom row log.
700 ' ------------------------------------------------------------------------------
710 '    Any triple-pair surviving the above tests gives rise to an
720 '    alphamagic square. Duplicate entries may occur, however, but
730 '    in that case it can be shown that CT = B(J,3)
740 ' ------------------------------------------------------------------------------
750      IF CT=B(J,3) THEN GOTO 950
760 ' ------------------------------------------------------------------------------
770 '    An advantage of the particular algorithm here employed is that
780 '    solutions are discovered in order of their Index No. We print
790 '    this, together with the solution (in standard normal form)
800 '    and its logorithm square alongside. TL = Top Left, etc.
810 ' ------------------------------------------------------------------------------
820      INDX=INDX+1: PRINT "No."INDX
830      TL=B(I,1): TR=B(J,3): BL=B(J,1): BR=B(I,3) 'shorthand
840      PRINT USING "####";TL,CT,TR,:PRINT  "  ",  'matrix formatting
850      PRINT USING "####";A(TL),A(CT),A(TR)       'matrix formatting
860      PRINT USING "####";CL,CEN,CR,:PRINT "  ",  'matrix formatting
870      PRINT USING "####";A(CL),A(CEN),A(CR)      'matrix formatting
880      PRINT USING "####";BL,CB,BR,:PRINT "  ",   'matrix formatting
890      PRINT USING "####";A(BL),A(CB),A(BR)       'matrix formatting
900      PRINT
910 ' ------------------------------------------------------------------------------
920 '    There may be still other squares with the same CENtre number.
930 '    If not, reset COUNT, take next case and search further.
940 ' ------------------------------------------------------------------------------
950      NEXT J: NEXT I
960      COUNT=0: NEXT CEN
970      PRINT "All possibilities up to" NMAX " examined"
980 ' ------------------------------------------------------------------------------
990 '    The first 110 English logorithms:
1000 ' -----------------------------------------------------------------------------
1010         DATA 4,3,3,5,4,4,3,5,5,4
1020         DATA 3,6,6,8,8,7,7,9,8,8
1030         DATA 6,9,9,11,10,10,9,11,11,10
1040         DATA 6,9,9,11,10,10,9,11,11,10
1050         DATA 5,8,8,10,9,9,8,10,10,9
1060         DATA 5,8,8,10,9,9,8,10,10,9
1070         DATA 5,8,8,10,9,9,8,10,10,9
1080         DATA 7,10,10,12,11,11,10,12,12,11
1090         DATA 6,9,9,11,10,10,9,11,11,10
1100         DATA 6,9,9,11,10,10,9,11,11,10
1120         DATA 10,13,13,15,14,14,13,15,15,14
1130         STOP
```

times its center number. Therefore, if the left-hand matrix is to be magic, the middle cell in its top row will have to contain (3×15) − (12+11) = 22. Similarly, if the square is to be *alpha*magic, the corresponding cell in the right-hand matrix will hold (3×7) − (6+6) = 9. Now, does *log* 22 = 9? Yes, it does. So far so good. Consider next the middle cell, right-hand column. Does *log*(45−29) = 21−14? Again, *yes*. Fine; next take the bottom row. Does *log*(45−37) = 21−16? *Yes!* This is too good to last; cross your fingers and try the last vacant cell. Does *log* (45−31) = 21−14? *No!* Yuck. . . .

So far, however, we have considered only the first pair of triples, and there remain nine other such combinations to be tried. A few minutes with pencil and paper will reward interested readers with a second alphamagic square, less elegant than the *Li shu* but still having 15 as its center num-

Continued on page 39

AlphaMagicSquaresOfOrder3

```
{------------------------------------------------------------------------------}
{ ==================== AlphaMagic Squares Detection ===================== }
{------------------------------------------------------------------------------}
{                          design : Lee Sallows                                }
{                  implementation : Victor Eijkhout                            }
{                                                                              }
{                          written in                                          }
{                             Turbo Pascal Version 3;                          }
{                             Borland International                            }
{------------------------------------------------------------------------------}

Program AlphaMagicSquaresOfOrder3;

Const Range=109;
   Logorithm : Array[0..Range] Of Integer
   = ( 4,3,3,5,4,4,3,5,5,4,
       3,6,6,8,8,7,7,9,8,8,
       6,9,9,11,10,10,9,11,11,10,
       6,9,9,11,10,10,9,11,11,10,
       5,8,8,10,9,9,8,10,10,9,
       5,8,8,10,9,9,8,10,10,9,
       5,8,8,10,9,9,8,10,10,9,
       7,10,10,12,11,11,10,12,12,11,
       6,9,9,11,10,10,9,11,11,10,
       6,9,9,11,10,10,9,11,11,10,
       10,13,13,15,14,14,13,15,15,14);

Var Square : Array[-1..1 , -1..1] Of Integer;

Var center,counter : Integer;

Function Min( x,y : Integer ):Integer;
Begin If x<y Then Min:=x Else Min:=y End;

{------------------------------------------------------------------------------}
{ ====================== output of completed square ==================== }
{------------------------------------------------------------------------------}
Procedure ReportAlphaMagicSquare( c,d1,d2, t,l,r,b : Integer );

 Var i,j : Integer;

Begin
  Square[-1,0]:=l; Square[1,0]:=r; Square[0,-1]:=b; Square[0,1]:=t;
  Square[-1,-1]:=c-d2; Square[1,1]:=c+d2;
  Square[-1,1]:=c-d1; Square[1,-1]:=c+d1;
  Square[0,0]:=c;
  Writeln('  alphamagic square No. ',counter);
  For i:=-1 To 1
  Do Begin For j:=-1 To 1
           Do Write( Square[j,i]:6 );
           Write(          <->     ');
           For j:=-1 To 1
           Do Write( Logorithm[ Square[j,i] ]:6 );
           Writeln(' ')
     End;
  Writeln(' ')
End; {======= procedure ReportAlphaMagicSquare ===========}
```

```
{-----------------------------------------------------------------------------}
{======================= test alphamagicality =====================}
{-----------------------------------------------------------------------------}
Procedure MaybeAlphaMagicSquare( cen,dist1,dist2 : Integer );

  Var MagicConstant, AlphaMagicConstant,
      left,right,top,bottom : Integer;

  Function MagicTriple( x,y : Integer; Var mid : Integer): Boolean;
  Begin MagicTriple := False;
        mid := MagicConstant -x-y;
        If ( mid>0 ) And ( mid<=Range ) And ( mid <> y )
          { the third test eliminates trivial solutions }
        Then MagicTriple := ( AlphaMagicConstant =
                               Logorithm[ x]
                             + Logorithm[ mid ]
                             + Logorithm[ y ]   )

  End;

 Begin
  MagicConstant := 3*cen;
  AlphaMagicConstant := 3*Logorithm[ cen ];
  If MagicTriple( cen-dist1,cen-dist2, left )
  Then If MagicTriple( cen-dist1,cen+dist2, top )
       Then If MagicTriple( cen+dist1,cen-dist2, bottom )
            Then If MagicTriple( cen+dist1,cen+dist2, right )
                 Then Begin counter:=counter+1;
                            ReportAlphaMagicSquare( cen,dist1,dist2,
                                                     top,left,right,bottom )
                      End
 End; {======= procedure MayBeAlphaMagicSquare =========}

{-----------------------------------------------------------------------------}
{======================= generate squares around =====================}
{======================= a given center number ======================}
{-----------------------------------------------------------------------------}
Procedure GenerateSquaresAroundCenter( c : Integer );

  Var k,l : Integer;

  Function LogoArithmeticTriple( cen,dist : Integer ): Boolean;
  Begin LogoArithmeticTriple :=
         Logorithm[ cen ] - Logorithm[ cen-dist ]
                          =
         Logorithm[ cen+dist ] - Logorithm[ cen ]
  End;

Begin
 For k:=Min( c,Range-c) DownTo 1
 Do If LogoArithmeticTriple( c,k )
    Then For l:=Min( c,Range-c) DownTo k+1
         Do If LogoArithmeticTriple( c,l )
            Then MaybeAlphaMagicSquare( c,k,l )
End; {==== procedure GenerateSquaresAroundCenter =========}

{-----------------------------------------------------------------------------}
                    Begin {======Main Program========}
                      counter:=0;
                      For center:=4 To Range-4
                      Do GenerateSquaresAroundCenter( center )
                    End.  {======Main Program========}
{-----------------------------------------------------------------------------}
```

Alphamagic Squares Nos. 1–10

Index Numbers	Alphamagic Squares				→		Logorithm Squares		
No. 1 (the *Li shu*)	5	22	18	five	twenty-two	eighteen	4	9	8
	28	15	2	twenty-eight	fifteen	two	11	7	3
	12	8	25	twelve	eight	twenty-five	6	5	10
No. 2	8	19	18	eight	nineteen	eighteen	5	8	8
	25	15	5	twenty-five	fifteen	five	10	7	4
	12	11	22	twelve	eleven	twenty-two	6	6	9
No. 3	15	72	48	fifteen	seventy-two	forty-eight	7	10	10
	78	45	12	seventy-eight	forty-five	twelve	12	9	6
	42	18	75	forty-two	eighteen	seventy-five	8	8	11
No. 4	18	69	48	eighteen	sixty-nine	forty-eight	8	9	10
	75	45	15	seventy-five	forty-five	fifteen	11	9	7
	42	31	72	forty-two	thirty-one	seventy-two	8	9	10
No. 5	21	66	48	twenty-one	sixty-six	forty-eight	9	8	10
	72	45	18	seventy-two	forty-five	eighteen	10	9	8
	42	24	69	forty-two	twenty-four	sixty-nine	8	10	9
No. 6	4	101	57	four	one hundred one	fifty-seven	4	13	10
	107	54	1	one hundred seven	fifty-four	one	15	9	3
	51	7	104	fifty-one	seven	one hundred four	8	5	14
No. 7	44	61	57	forty-four	sixty-one	fifty-seven	9	8	10
	67	54	41	sixty-seven	fifty-four	forty-one	10	9	8
	51	47	64	fifty-one	forty-seven	sixty-four	8	10	9
No. 8	5	102	58	five	one hundred two	fifty-eight	4	13	10
	108	55	2	one hundred eight	fifty-five	two	15	9	3
	52	8	105	fifty-two	eight	one hundred five	8	5	14
No. 9	45	62	58	forty-five	sixty-two	fifty-eight	9	8	10
	68	55	42	sixty-eight	fifty-five	forty-two	10	9	8
	52	48	65	fifty-two	forty-eight	sixty-five	8	10	9
No. 10	46	78	101	forty-six	seventy-eight	one hundred one	8	12	13
	130	75	20	one hundred thirty	seventy-five	twenty	16	11	6
	49	72	104	forty-nine	seventy-two	one hundred four	9	10	14

Figure 12. **The first ten English alphamagic squares of order 3, together with their logorithm squares.**

ber. Try it; one doing is worth a hundred seeings (old Northumbrian proverb). But what about all the other possible center numbers? To canvass all cases systematically, we need to begin with $C = 4$ (a lower number would be pointless, at least 4 distinct triples being demanded in any square), considering in turn $C = 5$, $C = 6, \ldots$, for as long as we wish to pursue the problem. Clearly, if ever a task was made for a computer, this is it.

The algorithm sketched above represents just one possible method, here incorporated into the simple Basic program labeled ALPHA.BAS; a Pascal form, AlphaMagicSquaresOfOrder3, was later prepared by my colleague Victor Eijkhout (see pages 34–37). Once the program was running, I was able to amuse myself over several weeks by exploring the alphamagic realm of order 3. It is a pursuit I can recommend to others. As one proceeds, the impression slowly grows of having ventured into a space offering almost unlimited recreational potential.

Besides the two examples already signalled, are there many other 3×3 English alphamagic squares? The answer is yes—an infinity of them. To see why, consider what happens if each of the *Li shu* entries is prefixed with the words *one hundred*. The addition of a uniform constant to both numbers (100) and logorithms (10) means that the resulting matrix (Figure 11) will again be alphamagic.

Such a square forms an example of what I call the *second harmonic* of the fundamental (first harmonic) square. Using *two hundred* instead of *one hundred* would result in the third harmonic, and so on. Subharmonics ("*zero point . . .*") are conceivable too, if a little far-fetched. The harmonic phenomenon thus gives rise to an endless progression of alphamagic squares, none of them claiming our serious further interest (save perhaps in specialized contexts) when once their fundamentals have been identified. What about the latter?

Figure 12 presents (in numerical form) the first ten English alphamagic squares of order 3; rotations and reflections of the same square are counted identical. Alphamagics using repeated numbers I deem trivial; repetitions in their logorithm squares (shown alongside) are not. The ten are put in sequence firstly by magic constant, which for order 3 is equivalent to ranking by center number, and secondly by the lowest number occurring: 2 in the first square, 5 in the second, and so on. Extendable to higher orders, this system attaches a unique index number to every square, thus providing a convenient method of reference. Where the lowest numbers of different squares coincide, ranking will depend on the second lowest, and so on. As with ordinary magic squares, standard practice is to reproduce examples so that the smallest corner number appears in the top left-hand position, with the smaller of its two immediate neighbors oriented to the top row (middle cell). Where different squares employ identical numbers, as may occur with higher orders, this latter convention will determine rank.

Looking over the list, certain characteristic features emerge. As

Alphamagic Square No. 91

215	372	298
378	295	212
292	218	375

Logorithms of No. 91

17	22	21
24	20	16
19	18	23

Figure 13

Alphamagic Square No. 120

249	320	328
378	299	220
270	278	349

Logorithms of No. 120

19	18	23
24	20	16
17	22	21

Figure 14

Latin Square

A	*B*	*C*
B	*C*	*A*
C	*A*	*B*

Figure 15

Greco-Latin Square

$A+a$	$B+b$	$C+c$
$B+c$	$C+a$	$A+b$
$C+b$	$A+c$	$B+a$

Figure 16

Log [No. 7]

nine	eight	ten
ten	nine	eight
eight	ten	nine

Figure 17

Log[Log[No. 7]]

4	5	3
3	4	5
5	3	4

Figure 18

Self-Reproducing Square

4	4	4
4	4	4
4	4	4

Figure 19

intuition might have led one to surmise, No. 1, the primordial Anglo-Saxon square, being the smallest and simplest (as well as oldest) exemplar in the language, is indeed none other than the *Li shu,* the Arkenstone among alphamagic gems, unmatched in revealing consecutive, minimal logorithms. Aside from its harmonics (Nos. 17, 26, 126, . . .), we have to ascend to the 91st square (magic constants = 885;60) before finding another consecutive specimen (Figure 13). There is only one other such fundamental square among the 217 alphamagics constructible from the English number-names up to *five hundred,* No. 120 (magic constants = 897;60) (Figure 14).

As we see, increasing word length entails that neither of these, nor in fact any beyond, are minimal. In our language, therefore, this is the exclusive property of the *Li shu.* The spirit of the yew tree knew well its errand to King Mī.

Glancing next at Square No. 7 (Figure 12), one detects the essential structure underlying the formation of 3×3 alphamagic squares: the well-known mathematical structure known as the *greco-latin* or *Eulerian* square. By a *latin* square of order N we mean one having N^2 entries of N different elements, none of them occurring twice in any row or column (Figure 15). A *greco-latin* square is one formed by superimposing two suitable latins such that each cell becomes occupied by a *distinct* entry. The term *greco-latin* derives from the once-common practice of using Greek and Roman letters to distinguish their two components; squares of this kind were first investigated in the 1770s by the great mathematician Leonhard Euler. It is easy to prove that order 3 admits of just one possibility—the square shown as Figure 16. (In combining a pair of latins it is not essential to *add* their separate elements, as is done here, but merely to append the contents of corresponding cells.)

Comparing this with Square No. 7 (among others, see also Nos. 1, 3, 6, 8, 9), the identity of form is immediately apparent (you will note the correspondence: $A \leftrightarrow 4$, $B \leftrightarrow 6$, $C \leftrightarrow 5$, $a \leftrightarrow 4$, $b \leftrightarrow 1$, $c \leftrightarrow 7$). Rows and columns (but not diagonals) in numerical representations of these squares are therefore composed of different permutations of the same set of *digits.* I leave it to readers to show that if $a = (b+c)/2$ and $C = (A+B)/2$ (the conditions necessary for magic diagonals), the resulting matrix is isomorphic with Lucas's formula. We shall have more to discuss about greco-latins later.

Staying with Square No. 7 for a moment, observe that the distribution of 1s, 4s, and 7s in the units' position of every entry has a curious consequence. Due to the chance that *log* 1 = *log* 2, *log* 4 = *log* 5, and *log* 7 = *log* 8, adding 1 to every number in the matrix results in a second alphamagic square: No. 9. Squares Nos. 6 and 8 form a similar related dyad. There are sixteen of these pairs—some adjacent, some more widely separated—among the first 100 squares.

The alphamagic properties of Square No. 7 are not yet entirely exhausted. Although trivial, the magic (latin) square formed by its logorithms (which I shall designate by *log* [No. 7]) is worth a closer look. Writing out *log* [No. 7] in full, we have Figure 17 as a result. Viewed thus, a natural question arises: could *log* [No. 7] by any chance be alphamagic, albeit trivial, too? The answer, of course, is yes, the magic constant of *log*[*log*[No. 7]] being 12 (see Figure 18).

At this point it is difficult not to wonder whether this second latin (magic) square is in turn alphamagic itself. Alas, repetition of the same process yields only a semi-magic derivative. Leaving apart superficial cases where the initial logorithm square is made up of nine identical numbers (a far from uncommon occurrence), I have been unable to find any such instance among the first few hundred English squares. No. 7 shares its distinction with Nos. 5, 9, and 36.

There is an interesting computer project here that ambitious readers may like to follow up. Ideally, of course, we seek a square giving rise to an unbroken chain of alphamagic derivatives, culminating, as any chain eventually must do, in a closed loop. The shortest and most elegant such alphamagic loop would be a self-reproducing square—Figure 19. I leave more complicated loops to the contemplation of interested parties. Lest the ground to be explored here seem unduly narrow, bear in mind that we are under no compunction to remain in the same language at each stage in the derivation process. What, for instance, might be the longest chain of *multilingual* alphamagic links constructible? In any case, the search for ever more potent magic "spells" of this and other kinds soon encourages a glance beyond the confines of English.

French Square No. 14

Quinze (15;6)	*Deux cent six* (206;11)	*Cent quinze* (115;10)
Deux cent douze (212;13)	*Cent douze* (112;9)	*Douze* (12;5)
Cent neuf (109;8)	*Dix huit* (18;7)	*Deux cent neuf* (209;12)

Figure 20

German Square No. 72

Fünfundvierzig (45;14)	Zweiundsechzig (62;14)	Achtundfünfzig (58;14)
Achtundsechzig (68;14)	Fünfundfünfzig (55;14)	Zweiundvierzig (42;14)
Zweiundfünfzig (52;14)	Achtundvierzig (48;14)	Fünfundsechzig (65;14)

Figure 21

Exotic Squares

The exact number of alphabetic languages used throughout the world has perhaps never been estimated. Clearly there are many. Besides those like our own employing Roman letters, there remain others using the Greek, Hebrew, and Cyrillic alphabets. The work of collecting and collating alphamagic squares in the various tongues and dialects opens a wide (if decidedly recondite) area of research. One has only to think of the enormous literature on ordinary magic squares, with its endless refinements and ramifications, almost all of which become reapplicable to alphamagic squares, to catch a glimpse of the undeveloped possibilities. My own peregrinations in the field having been superficial, I shall present here only a few examples of order 3.

Investigating 3×3 alphamagic squares in different languages calls for no alterations to the program already described, save in loading appropriate logorithm data into memory. Having had some experience in this line of late, I can report that ascertaining the correct spelling of foreign cardinals is often trickier than one suspects. Books supposedly supplying this information should be treated circumspectly (in French, is 101 *cent un* or *cent et un*?). Typing in word lengths without introducing errors is another task requiring perseverance and concentration; a subprogram for calculating letter-counts from the words themselves is advisable. Without care in this preparatory phase, interpretation of the printout is troubled with doubts.

Taking French as an initial object of study, I was intrigued to discover only a single alphamagic square using number-names in the range up to *deux cents* (200). It seemed that Gallic orthography combined with a vigesimal (twenty-based) system of counting to produce singular effects on the alphamagic plane. Thinking what a rare collector's item this must represent if it turned out to be the sole existing French alphamagic square of order 3, I quickly extended the search up to *trois cents*, only to be glutted with a sudden deluge of 225 new specimens! Square No. 14 (magic con-

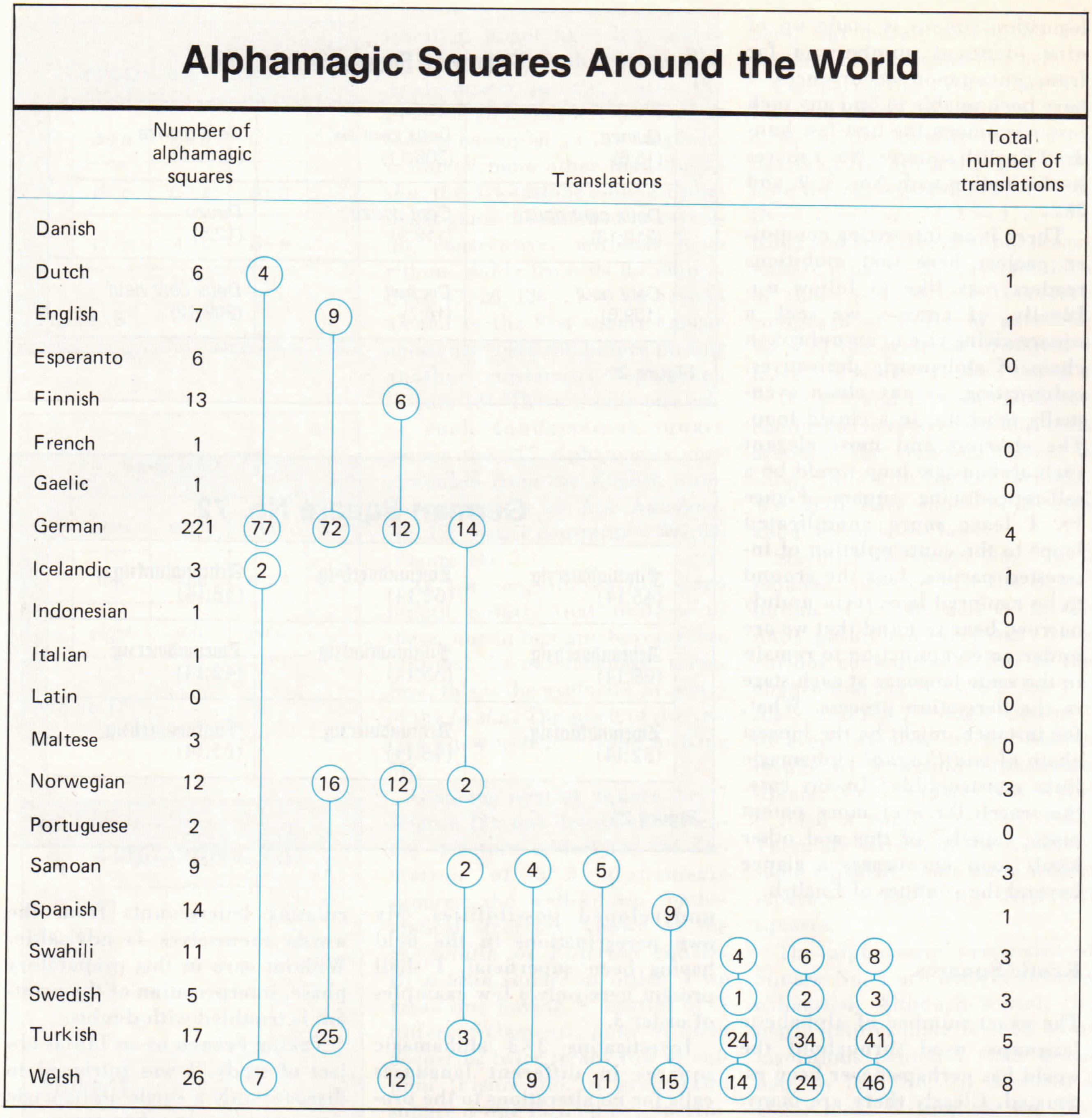


***Figure 22.* What is the total number of alphamagic squares with cardinals not higher than 100? This chart shows the answer for squares in different languages (left). In the column marked "translations," a circled figure is the index number of a given square, with lines linking numerically identical squares (mutual translations). Thus, Dutch Square No. 4 is a translation of German Square No. 77.**

stants = 336;27) was the first (of 3) to show consecutive logorithms (Figure 20). [In parentheses, the numerical representation of each number-word is followed by the *log* or letter-count.]

A curiosity worth remarking is the prevalence of prime numbers among French alphamagics, a by-product of frequent *un*, *trois*, and *sept* terminations. Even so, a square composed uniquely of primes, in this or in any other language, has yet to be identified. The urge to uncover specialist items of this kind will probably prove a stimulus to logophiles for some time to come. Serious aficionados will hardly rest until the Tower of Babel has been ransacked from roof to basement.

Following the French experience, I was better prepared for a foray into German. After entering the new logorithms and typing RUN, within seconds the printer chirped into life and began spitting out alphamagic squares in a steady rhythmical tattoo evocative of massed hordes on the march.

The reason for this regularity was soon apparent: every one of the 221 squares resulting from number-names under *hundert* (100) employs nine double-digit numbers; with few exceptions, the adjacently printed logorithms of every one of these nine were the same: 14.

Many readers, I imagine, will be surprised to learn of hundreds of alphamagic squares extant in three different languages. How is this prodigality made possible? The answer lies, simply enough, in the (inevitable) regularity of our naming systems for cardinals higher than *twenty*, the designations beyond this point being exact verbal counterparts of their decimal-digit representations (*twenty-one* = 20 + 1, *twenty-two* = 20 + 2, and so on). Thus, the combinative properties of numbers are often paralleled in their logorithms, with the result that many an unexceptional magic square (of which there are myriads, contrary to expectation), is automatically rendered alphamagic. In German—an extreme case, where the words for 1, 2, 3, 4, 5, 8, and 9 all have four letters, and those for 20, 30, 40, 50, 60, 70, 80, 90, and 100 all have seven—this factor issues in a rash of uniform logorithm squares, few of them revealing any redeeming feature of interest. A typical example is No. 72, shown in Figure 21 (magic constants = 165;42).

The trouble with squares generated by this parallel effect is their structural transparency, which robs them of logological charm. As logophiles we prize cunning arrangements exploiting unsuspected linguistic fortuity. In almost any language, therefore, the vast majority of squares will fail to command admiration. In general, of course, as in Gardner's marvellous anagram prefacing this article, alphamagic elegance resides in small numbers.

Wearying of pedestrian languages, I turned next to some of the less familiar tongues. Keeping research within manageable bounds, surveys were limited to cardinals in the range up to 100. Figure 22, a recherché anthology if ever there was one, records the numbers of squares discovered in each case. Totals are generally modest, which is not to say they would remain so if the census were extended further. Raising the ceiling to 200, for instance, second harmonics will account for a doubling in figures,. at the very least.

A study of squares in foreign languages can hardly proceed very far before an obvious contingency springs to thought. Has anyone noticed, I wonder, that the German square No. 72 given above is a perfect *translation* of English square No. 9? As a matter of fact, both the German translation of $\log_e$ [No. 9] and the English translation of $\log_g$ [No. 72] are themselves alphamagic, like their originals; but here we are straying into a less central, even frivolous hinterland. Once glimpsed, of course, the notion of such a (primary) correspondence soon urges systematic comparison among squares, alphamagic translations forming yet another branch to explore in the logological labyrinth.

Latin Square No. 4

SEPTEM ET NONAGINTA (XCVII;XVII)	CENTUM SEPTUAGINTA QUINQUE (CLXXV;XXIV)	CENTUM TRIGINTA NOVEM (CXXXIX;XIX)
CENTUM SEPTUAGINTA NOVEM (CLXXIX;XXII)	CENTUM TRIGINTA SEPTEM (CXXXVII;XX)	QUINQUE ET NONAGINTA (XCV;XVIII)
CENTUM TRIGINTA QUINQUE (CXXXV;XXI)	NOVEM ET NONAGINTA (XCIX;XVI)	CENTUM SEPTUAGINTA SEPTEM (CLXXVII;XXIII)

Figure 23

Italian Square No. 3

Ottantasette (87;12)	*Cento sessantacinque* (165;19)	*Cento ventinove* (129;14)
Cento sessantanove (169;17)	*Cento ventisette* (127;15)	*Ottantacinque* (85;13)
Cento venticinque (125;16)	*Ottantanove* (89;11)	*Cento sessantasette* (167;18)

Figure 24

Alphamagic Translations between Swedish and Swahili

Swedish No. 1 ⟷ Swahili No. 4

Fyrtiofem (45;9)	Sextiotre (63;9)	Femtiosju (57;9)
Sextiosju (67;9)	Femtiofem (55;9)	Fyrtiotre (43;9)
Femtiotre (53;9)	Fyrtiosju (47;9)	Sextiofem (65;9)

Arobaini na tano (45;14)	Sitini na tatu (63;12)	Hamsini na saba (57;13)
Sitini na saba (67;12)	Hamsini na tano (55;13)	Arobaini na tatu (43;14)
Hamsini na tatu (53;13)	Arobaini na saba (47;14)	Sitini na tano (65;12)

Swedish No. 2 ⟷ Swahili No. 6

Fyrtiosex (46;9)	Sextiotre (63;9)	Femtionio (59;9)
Sextionio (69;9)	Femtiosex (56;9)	Fyrtiotre (43;9)
Femtiotre (53;9)	Fyrtionio (49;9)	Sextiosex (66;9)

Arobaini na sita (46;14)	Sitini na tatu (63;12)	Hamsini na tisa (59;13)
Sitini na tisa (69;12)	Hamsini na sita (56;13)	Arobaini na tatu (43;14)
Hamsini na tatu (53;13)	Arobaini na tisa (49;14)	Sitini na sita (66;12)

Swedish No. 3 ⟷ Swahili No. 8

Fyrtiofem (45;9)	Sextiosju (67;9)	Femtionio (59;9)
Sjuttioen (71;9)	Femtiosju (57;9)	Fyrtiotre (43;9)
Femtiofem (55;9)	Fyrtiosju (47;9)	Sextionio (69;9)

Arobaini na tano (45;14)	Sitini na saba (67;12)	Hamsini na tisa (59;13)
Sabini na moja (71;12)	Hamsini na saba (57;13)	Arobaini na tatu (43;14)
Hamsini na tano (55;13)	Arobaini na saba (47;14)	Sitini na tisa (69;12)

Figure 25. **Swedish alphamagics Nos. 1, 2, and 3 translate into Swahili alphamagics Nos. 4, 6, and 8. Word-game players will note that in Swahili, *sita* (6) is an anagram of *tisa* (9).**

Figure 22 includes a résumé of the interlingual connections so far established.

Two of the languages listed show no alphamagic squares at all in the range investigated. Extending examination of the first of these discovers six Danish squares using numbers below *tohundrede* (200). Likewise, in the second case, four squares are brought to light, No. 4 (magic constants = 411;60) being a rare consecutive-logorithm curio using odd numbers only (Figure 23). Here the influence of underlying latin squares is unmistakable. Likewise, early Roman influence is perhaps responsible for the consecutive logorithms to be found in Figure 24, a *modern* Italian ("I, a Latin") square—No. 3 (magic constants = 381;45). Note the constant difference between corresponding entries at both numerical and logorithm levels in this geographically related pair.

Oddly, of all the languages so

far examined, there is one which stands out as peculiarly rich in alphamagic *translations.* English is poor, yielding only the example previously cited. French, together with others, has none. Norwegian and Samoan show three, as do Swedish and Swahili, an alliterative duo remarkable in that Nos. 1, 2, and 3 in the former translate into Nos. 4, 6, and 8 in the latter (as shown in Figure 25). German yields no less than four, which is not surprising in view of its total of 221 squares. And Turkish delights in five, three of them correlating with squares in the most prolific source of all: it is the language of the West Britons, the language of the Bards, Welsh.

Together with its sister tongues Breton and Cornish, Welsh belongs to the Celtic family of languages, which includes Irish, Manx, and Gaelic. In former times the vigesimal system was current, but except in reading the clock, present-day Welsh has replaced this with decimal usage. Whether the originators of this reform had any premonition of its alphamagic consequences must remain conjectural, but the effects have been remarkable indeed. Old-fashioned (vigesimal) Welsh, which I have also examined up to *cant* (100), manifests no alphamagic squares whatever. Modern Welsh, on the other hand, rejoices in twenty-six squares in this range, no less than eight of them corresponding to translations of squares in either Turkish (3 cases), Samoan (2 cases), Spanish, Icelandic, or Norwegian. The latter instance furnishes a striking consecutive-logorithm cameo using even numbers only (magic constants = 216;33); see Figure 26.

Welsh Square No. 12

Chwech deg dau (62;12)	*Wyth deg* (80;7)	*Saith det pedwar* (74;14)
Wyth deg pedwar (84;13)	*Saith deg dau* (72;11)	*Chwech deg* (60;9)
Saith deg (70;8)	*Chwech deg pedwar* (64;15)	*Wyth deg dau* (82;10)

Norwegian Square No. 12

Sekstito (62;8)	*Åtti* (80;4)	*Syttifire* (74;9)
Åttifire (84;8)	*Syttito* (72;7)	*Seksti* (60;6)
Sytti (70;5)	*Sekstifire* (64;10)	*Åttito* (82;6)

Figure 26

Amazingly, as many as *six* of these twenty-six Welsh squares show consecutive logorithms, a staggering total considering that of the remaining 333 squares spread over twenty languages in Figure 22, there is but a single instance of another consecutive-logorithm square: the *Li shu* (the French, Latin, and Italian examples given earlier lying outside our two-digit range). None of the Cambrian six are minimal, however, the shortest number-name in Welsh containing two letters, while the smallest series of logorithms occurring runs from 7 up to 15.

A detailed treatment of the unnumbered curiosities and secondary correlations to be found among alphamagic squares across the different languages is beyond the scope of a single article. Leaving the field to enthusiasts who may like to pursue these researches—seeking, perhaps, what I failed to discover, a *triple*-language alphamagic translation—it is time to return to English and a look at the higher orders.

What happens in going beyond 3×3 specimens to larger squares? Is the way strewn with gems "beyond the wildest fantasies of logomania," or are these really all "just so much logological junk"? In Part II of this article, to appear in the next issue, investigation into the higher orders turns up unexpected challenges to ingenuity, with intriguing sidelights on "normal" alphamagics and the role of "minimal formulae" in seeking solutions. Programmers with a taste for recreations are promised rich pickings in fresh realms of opportunity.

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